03. Kinetic theory Of Gases

Gas laws :-

Laws that relate the pressure, volume & temperature of the gas are known as gas laws.

Three basic gas laws,

1] Boyle's law.

2] Charle's law.

3] Gay-Lussac's law.

1] Boyle's law :-

For fixed mass of gas at constant temperature, Pressure is inversely proportional to volume.

mathematically it is written as,

$$\boxed{P \propto \frac{1}{V}}$$

where,

Temperature = constant

P = Pressure

V = volume

2] charle's law :-

The volume of a fixed mass of gas is directly proportional to its absolute temperature if the pressure is kept constant.

mathematically it is written as,

$$\boxed{V \alpha T}$$

p = constant

where,

V = volume

T = Temperature

3] Gay-Lussac's law :-

The pressure of a fixed mass of a gas is directly proportional to its absolute temperature if the volume is kept constant.

mathematically it is written as,

$$\boxed{P \alpha T}$$ Volume = constant

where,

P = Pressure

T = Temperature

Avogadro's number :-

The number of particles (atoms or molecules) in 22·4 litres of any gas is termed as Avogadro's number

Approximately taken as,

$$\boxed{N_A = 6.023 \times 10^{23}}$$

Mole :-

One mole is that amount of substance which contains exactly $6.02214076 \times 10^{23}$ molecules.

Elastic collision of particles :-

collision between particles in which both kinetic energy & momentum of particles are conserved is called as elastic collision.

3. KINETIC THEORY OF GASES

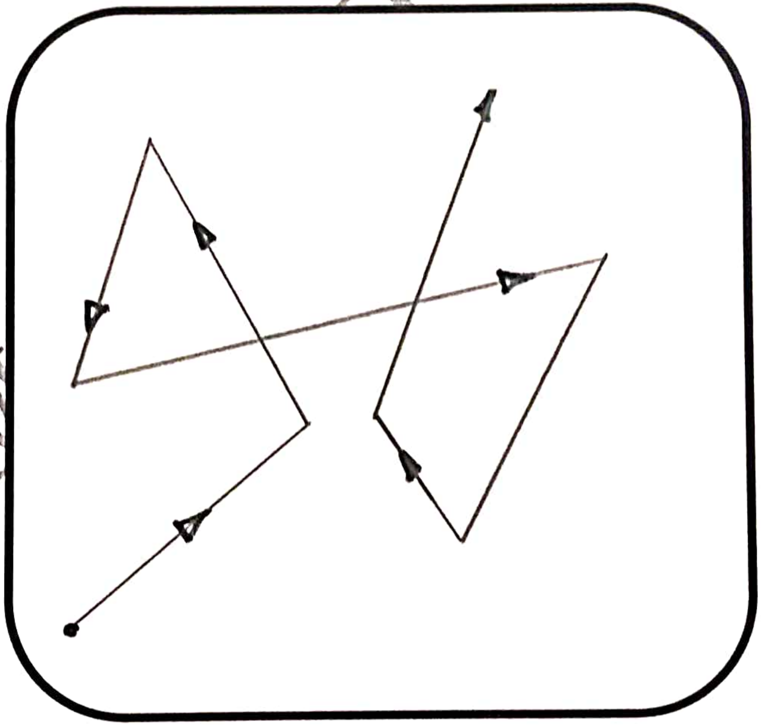
Dalton's law of partial pressures:-

The total pressure of a mixture of two or more non-reactive gases is the sum of the partial pressures of the individual gases in the mixture.

Ideal gas:-

A gas which obeys ideal gas equation at all pressures and temperatures is an ideal gas.

Mean Free Path:-



DIAGRAM

A Typical molecule in a gas executing random motion

3. KINETIC THEORY OF GASES

1. The typical path followed by a particle during Brownian motion as shown in figure.

2. When molecules approaches another molecule, there is a repulsive force between them, due to which the molecules behave as small hard spherical particles.

3. This leads to elastic collisions between the molecules.

4. Therefore both the speed and direction of motion of the molecules change abruptly.

5. The molecules also collide with the walls of the container.

6. Molecules exerts force on each other only during collisions. Thus in between two successive collisions the molecules move along straight paths with constant velocity.

7. The average distance traversed by a molecule with constant velocity between two successive collisions is called mean free path.

8. The mean free path varies,

a) Inversely with density of the gas.

Where,

N= Number of molecules

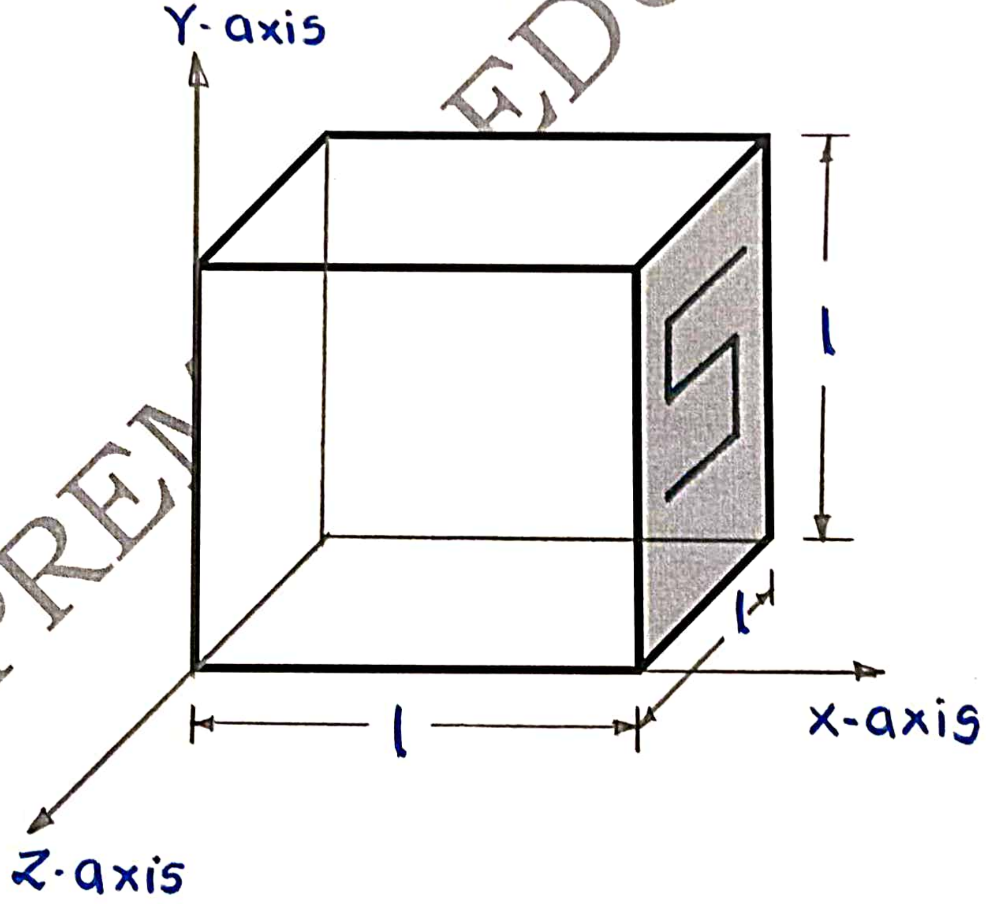
V= Volume of the gas

b) Inversely with square of the diameter of molecule d [Because it depends on a cross section of a molecule]

Mathematically it is written as,

$$\boxed{\lambda = \frac{1}{\sqrt{2}\,\pi d^2 (N|V)}}$$

PRESSURE OF IDEAL GAS:-



Consider an ideal gas filled in a cubical container of side 'l'

Where,

l = Side of cubical container

$A = l^2$ = Area of face of cube

$V = l^3$ = Volume of container

Volume of gas.

Where,

N = No. of molecules of the gas

m = mass of each gas of molecules.

M = Total mass of gas inside container.

$$\boxed{M = mN}$$

$$\boxed{\rho = \frac{M}{V}}$$

Density of the gas

Now, As there are N no. of gas molecules & 3-axes,

$\frac{N}{3}$ molecule will move along each axis

$\frac{N}{3}$ molecule will move along x-axis

$\frac{N}{3}$ molecule will move along Y-axis

$\frac{N}{3}$ molecules will move along z-axis

Now,

Consider one gas molecules of mass 'm'

move along x-axis with constant velocity,

Initial momentum before collision,

$$P_i = mv_1$$

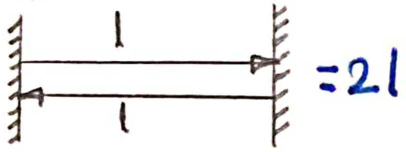
Final momentum after collision,

$$P_f = -mv_1$$

Change in momentum = Final momentum − Initial momentum

$$\text{Change in momentum} = -mv_1 - mv_1$$

$$dp = -2mv_1$$



Gain in momentum of surface S is,

$$dp = 2mv_1$$

Distance between two successive collision

is $= 2l$

where,

dt = Time interval between two successive collision.

$$\text{Velocity} = \frac{\text{Distance}}{\text{Time}}$$

$$V_1 = \frac{2l}{dt}$$

$$\boxed{dt = \frac{2l}{V_1}}$$

Rate of change of momentum of surface 'S' due to one molecule, $\frac{dp}{dt} = \frac{\cancel{2}mV_1}{\cancel{2}l/V_1}$

$$\boxed{\frac{dp}{dt} = \frac{mV_1^2}{l}}$$

Rate of change of momentum of surface 'S' due to N/3 molecules,

$$\frac{dp}{dt} = \frac{mV_1^2}{l} + \frac{mV_2^2}{l} + \cdots\cdots\cdots + \frac{mV_{N/3}^2}{l}$$

$$\frac{dp}{dt} = \frac{m}{l}\left(V_1^2 + V_2^2 + \cdots\cdots + V_{N/3}^2\right)$$

$$\frac{dp}{dt} = \frac{m}{l} \times \frac{N}{3} \times \left(\frac{V_1^2 + V_2^2 + \cdots\cdots + V_{N/3}^2}{N/3}\right)$$

$$\frac{dp}{dt} = \frac{1}{3}\frac{mN}{l} \times \overline{V}^2$$

$$\frac{dp}{dt} = \frac{1}{3} \times \frac{M}{l} \times \overline{V}^2$$

According to Newton's second law of motion, Rate of change of momentum is directly proportional to the applied force,

$$F = \frac{dp}{dt}$$

$$\boxed{F = \frac{1}{3} \times \frac{M}{l} \times \overline{V^2}}$$

$$\text{Pressure} = \frac{\text{Force}}{\text{Area}}$$

$$P = \frac{F}{A}$$

$$P = \frac{1}{3} \times \frac{M}{l} \times \frac{\overline{V^2}}{A}$$

$$P = \frac{1}{3} \times \frac{M}{l^3} \times \overline{V^2}$$

$$P = \frac{1}{3} \times \frac{M}{V} \times \overline{V^2}$$

$$P = \frac{1}{3} \times \rho \times \overline{V^2}$$

$$P = \frac{1}{3} \times \rho \left(\sqrt{\overline{V^2}}\right)^2$$

$$\boxed{P = \frac{1}{3} \times \rho V^2}$$

Prove that $V_{rms} \propto \sqrt{T}$

Pressure exerted by a gas molecules on the walls of container is,

$$P = \frac{1}{3} \rho V_{rms}^2$$

$$P = \frac{1}{3} \times \frac{M}{V} \times V_{rms}^2 \quad \cdots\cdots \left[\rho = \frac{M}{V}\right]$$

$$3PV = M V_{rms}^2$$

$$V_{rms}^2 = \frac{3PV}{M} \quad \text{-----}[1]$$

According to equation of state,

$$\boxed{PV = nRT}$$

Put, PV=nRT in equation [1],

$$V_{rms}^2 = \frac{3nRT}{M} \quad \cdots\cdots [PV = nRT]$$

$$n = \frac{\text{mass of the gas } (M)}{\text{mass of Avogadro's no. of molecules of gas } (M_0)}$$

$$n = \frac{M}{M_0} \qquad n = \frac{\cancel{m} \times N}{\cancel{m} \times N_A}$$

$$\boxed{n = \frac{N}{N_A}}$$

$$V_{rms}^2 = \frac{3 \times \cancel{n} RT}{N_A \times m \times \cancel{N}}$$

$$V_{rms}^2 = \frac{3RT}{M_0} \quad \ldots\ldots \quad [m \times N_A = M_0]$$

$$V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

$$\boxed{V_{rms} \propto \sqrt{T}}$$

Hence proved.

Average kinetic energy of gas molecules $\propto T$

Pressure exerted by a gas molecules on the walls of container is,

$$P = \frac{1}{3} \rho V_{rms}^2$$

$$P = \frac{1}{3} \times \frac{M}{V} \times V_{rms}^2 \quad \cdots \left[\rho = \frac{M}{V}\right]$$

$$3PV = MV_{rms}^2 \quad \cdots [1]$$

$\boxed{M = m \times N}$ Put in eq^n ①,

$$3PV = m \times N \times V_{rms}^2 \quad \cdots [2]$$

Put, $PV = nRT$ in eq^n ②,

$$3nRT = m \times N \times V_{rms}^2$$

$$mV_{rms}^2 = \frac{3nRT}{N}$$

$$mV_{rms}^2 = 3 \times \frac{N}{N_A} \times \frac{RT}{N} \quad \cdots \left[n = \frac{N}{N_A}\right]$$

Multiplication by $\frac{1}{2}$ on both sides,

$$\frac{1}{2} mV_{rms}^2 = \frac{3}{2} \times \frac{\cancel{N}}{N_A} \times \frac{RT}{\cancel{N}}$$

$$\text{Kinetic energy of gas molecules} = \frac{3}{2} \times \frac{RT}{N_A}$$

Kinetic energy of gas molecules $\propto T$

Prove that Boyle's law $P \propto \frac{1}{V}$

Temperature constant

$PV = \text{constant}$

where, P = Pressure

V = volume

T = Temperature

Pressure exerted by a gas molecules on the walls of container is,

$$P = \frac{1}{3} \rho \times V_{rms}^2.$$

$$P = \frac{1}{3} \times \frac{M}{V} \times V_{rms}^2 \ldots \left[\rho = \frac{M}{V}\right]$$

$$PV = \frac{1}{3} m \times N \times V_{rms}^2 \ldots \left[M = m \times N\right]$$

$$PV = \frac{1}{3} \times N \times m V_{rms}^2$$

Multiply & Divide By 2,

$$PV = \frac{2}{3} \times N \times \boxed{\frac{1}{2} m V_{rms}^2}$$

$$PV = \frac{2}{3} \times N \times \text{Average K.E. of gas molecules.}$$

$PV = \text{constant}$

$$\boxed{P \propto \frac{1}{V}}$$

$T = \text{constant}$

Hence, **Boyle's law is prooved.**

- K.E. per unit volume $= \frac{3}{2} P$

Pressure exerted by a gas molecules on the walls of container is,

$$P = \frac{1}{3} \rho \times V_{rms}^2$$

$$P = \frac{1}{3} \times \frac{M}{V} \times V_{rms}^2 \quad \left[\rho = \frac{M}{V}\right]$$

$$3P = \frac{m \times N}{V} \times V_{rms}^2 \quad [M = m \times N]$$

$$3P = \frac{N \times m V_{rms}^2}{V}$$

Multiply by $\frac{1}{2}$ on both sides,

$$\frac{3P}{2} = \frac{\boxed{N \times \frac{1}{2} m v_{rms}^2}}{V}$$

$$\frac{3P}{2} = \frac{\text{Avg. K.E. of gas}}{\text{Volume of gas}}$$

$$\boxed{\frac{\text{Avg. K.E. of gas}}{\text{volume of gas}} = \frac{3}{2} P}$$

03. kinetic Theory of Gases.

Avg. K.E. per molecule $= \frac{3}{2}kT$

RMS velocity of a gas molecule

$$V_{rms} = \sqrt{\frac{3RT}{M_0}}$$

molar mass

$$V_{rms}^2 = \frac{3RT}{M_0}$$

Avg. K.E. of a molecule of gas $= \frac{1}{2} m V_{rms}^2$

$$= \frac{1}{2} m V_{rms}^2$$

$$= \frac{1}{2} m \times \frac{3RT}{M_0}$$

$$= \frac{1}{2} \times \frac{\cancel{m} \times 3k\cancel{N_A} \times T}{\cancel{m} \times \cancel{N_A}}$$

Avg. K.E. of a molecule of gas $= \frac{1}{2} \times 3kT$

$$\boxed{\text{Avg. K.E. per molecule} = \frac{3}{2}kT}$$

03. Kinetic Theory of Gases

Avg. K.E. per mole $= \frac{3}{2} RT$

1 mole of gas consist of Avogadro's no. of molecules i.e. NA molecules

K.E. of 1 mole of gas = K.E. of Avogadro's no of molecules.

K.E. of 1 mole of gas = NA x Avg. K.E. of 1 molecules of gas.

$$= N_A \times \frac{3}{2} kT$$

$$= \cancel{N_A} \times \frac{3}{2} \times \frac{R}{\cancel{N_A}} \times T$$

K.E. of 1 mole of gas. $= \frac{3}{2} \times T \times R$

$$\text{Avg. K.E./mole} = \frac{3}{2} RT$$

$$\boxed{k = \frac{R}{N_A}}$$

Boltzmann constant.

$R = 8.314 \frac{J}{mol\,k}$

$k = 1.38 \times 10^{-23} \frac{J}{k}$

3. KINETIC THEORY OF GASES

PERFECT BLACK BODY

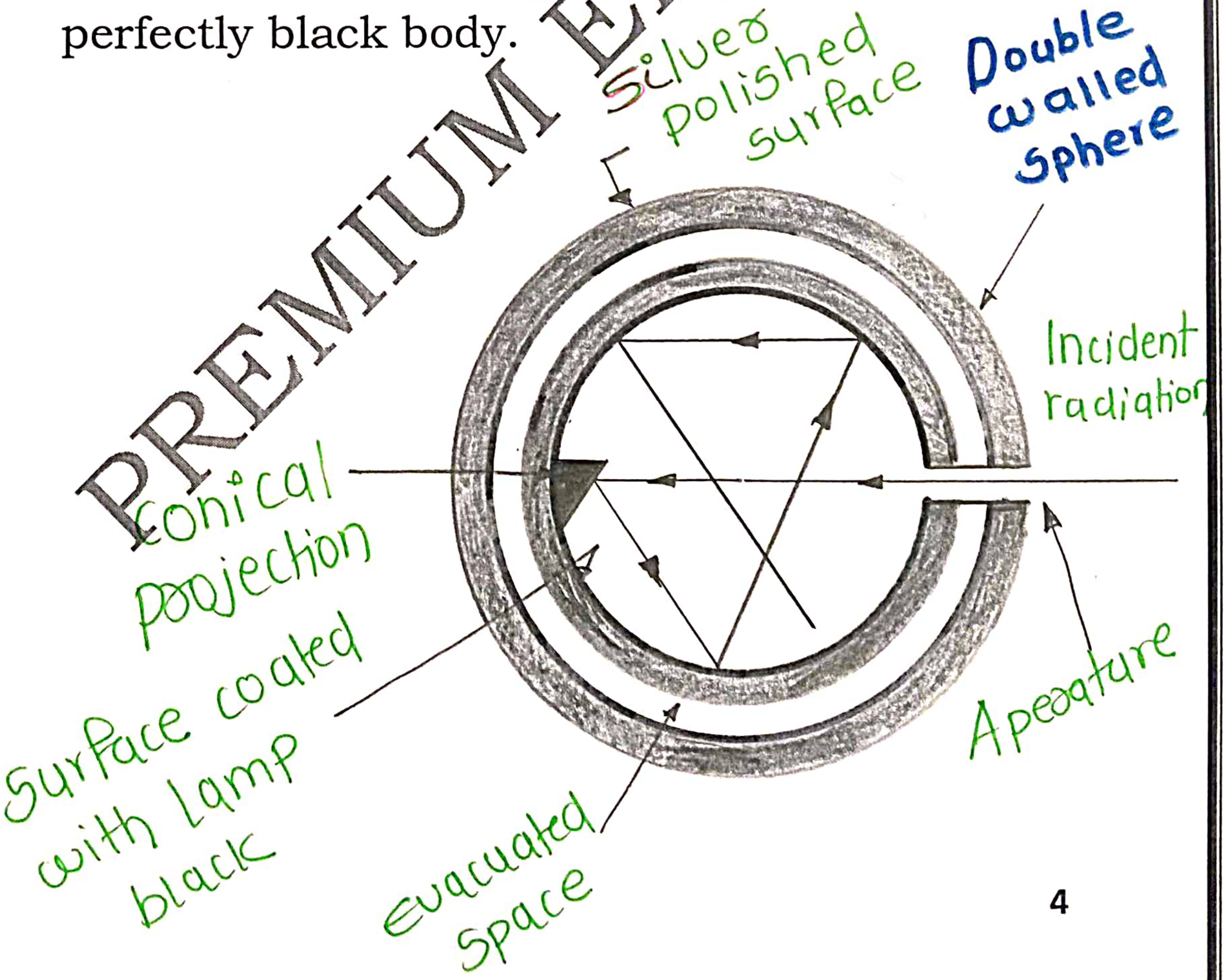
1. A body which absorbs the entire radiant energy incident on it, is called as ideal or perfect blackbody.

2. Perfectly black body does not exist in nature.

3. **Examples :-** Lamp black, Platinum black

Ferry's perfectly black body:-

With neat labeled diagram, explain Ferry's perfectly black body.



3. KINETIC THEORY OF GASES

1. Ferry's perfectly black body consist of, Double walled hollow sphere having tiny hole or aperture, through which radiant heat can enter.
2. The space between the walls is evacuated and outer surface of the sphere is silvered.
3. The inner surface of sphere is coated with lamp black.
4. There is conical projection on the inner surface of sphere opposite the aperture.
5. The projection ensures that a ray travelling along the axis of the aperture is not incident normally on the surface and is therefore not reflected back along the same path.
6. A heat ray entering the sphere through the aperture suffers multiple reflections and is almost completely absorbed inside.
7. Thus, the aperture behaves like a perfect blackbody.
8. The effective area of perfectly black body is equal to the area of the aperture.

3. KINETIC THEORY OF GASES

State Prevost's theory of exchange of heat

All bodies at all temperatures above 0 K [absolute zero temperature] radiate thermal energy and at the same time, they absorb radiation received from the surroundings.

Radiation:-

It is the process of transfer of heat from a body which is at higher temperature to a body which is at lower temperature in the form of electromagnetic waves.

Electromagnetic waves:-

1] Infrared rays

2] Gamma rays

3] UV Rays

4] Light rays

5] X-Rays

6] Visible spectrum

Properties:

1] The heat energy which is transferred is called as radiant heat or radiant energy.

2] This process of transfer of heat is radiation takes place at all temperature except at absolute zero temperature.

3] It is the fastest mode of transfer of heat.

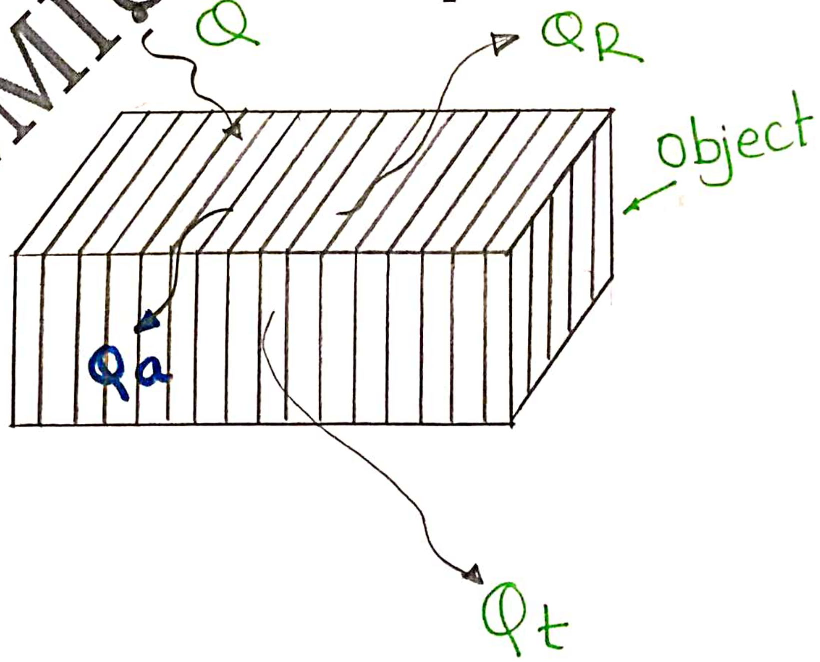
4] EM waves i.e. heat radiations travel with the speed of light in air.

5] Heat radiation can pass through vaccum.

6] When heat radiation travel through medium particle of medium does not get heated.

7] EM waves consist of X-rays, Gamma-Rays, UV rays, Visible spectrum rays, Infrared rays.

8] Radiation in IR spectrum are mostly thermal radiations due to temperature of body.



where,

Q = Amount of radiant heat which is emitted by source.

Q_a = Amount of radiant heat absorbed by the body.

Q_r = Amount of radiant heat reflected by the body

Q_t = Amount of radiant heat transmitted by the body.

According to law of conservation of energy,

$$\boxed{Q = Q_a + Q_r + Q_t}.$$

Divided by Q, above eq^n.

$$\frac{\cancel{Q}}{\cancel{Q}} = \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q}$$

$$1 = \frac{Q_a}{Q} + \frac{Q_r}{Q} + \frac{Q_t}{Q}$$

$$\boxed{a = \frac{Q_a}{Q}} \quad \boxed{r = \frac{Q_r}{Q}} \quad \boxed{t = \frac{Q_t}{Q}}$$

where,

Q = Amount of radiant heat which is emitted by source.

Q_a = Amount of radiant heat absorbed by the body.

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$$\boxed{a = \frac{Q_a}{Q}} \quad \boxed{r = \frac{Q_r}{Q}} \quad \boxed{t = \frac{Q_t}{Q}}$$

3. KINETIC THEORY OF GASES

$$\boxed{1 = a + r + t}$$

If $a = 1$ perfectly black body

$r = 1$ polished surface

$t = 1$ Glass / quartz.

$$\boxed{a + r + t = 1}$$

$$a = 1$$

$$1 + r + t = 1$$

$$\boxed{r + t = 0}$$

$$r = 1$$

$$a + 1 + t = 1$$

$$\boxed{a + t = 0}$$

$$a + r + t = 1$$

$$\boxed{t = 1}$$

$$a + r + 1 = 1$$

$$\boxed{a + r = 0}$$

1] **Coefficient of absorption (a)**

– It is the ratio of amount of radiant heat absorbed by the body to the total amount of radiant heat incident upon it.

It is denoted by 'a'

It is also called as absorptive power or absorptivity.

$$\boxed{\frac{Q_a}{Q} = a}$$

2] **Coefficient of reflection(r)**

It is the ratio of amount of radiant heat reflected by the body to the total amount of radiant heat incident upon it.

$$\boxed{r = \frac{Q_r}{Q}}$$

3] **Coefficient of transmission (t)**

It is the ratio of amount of radiant heat transmitted by the body to the total amount of radiant heat incident upon it.

$$\boxed{t = \frac{\varphi_t}{\varphi}}$$

Define emissive power of the body:-

The quantity of heat radiated per unit area per unit time is defined as emissive power of the body at given temperature.

$$\boxed{R = \frac{\varphi}{At}}$$

S.I. unit of emissive power $\frac{J}{m^2 S}$ or

W/m^2.

Dimensions of emissive power,

$$[L^0 M^1 T^{-3}]$$

Coefficient of emission:- (e)

The coefficient of emission or emissivity (e) of a given surface is the ratio of the emissive power R of the surface to the emissive power R of a perfect black surface, at the same temperature.

$$\boxed{e = \frac{R}{R_B}}$$

For perfect black body $e = 1$

For perfect reflector $e = 0$

For an ordinary body $0 < e < 1$ depending on the nature of the surface.

State Wien's displacement law:-

The wavelength for which emissive power of a blackbody is maximum is inversely proportional to the absolute temperature of the blackbody.

$\boxed{\lambda_{max} \propto \frac{1}{T}}$ $\lambda_{max} \propto \frac{b}{T}$

$\boxed{\lambda_{max} T = b}$ where, b is called as wiens constant

The value is 2.897×10^{-3} mK.

State Stefan-Boltzmann law

The rate of emission of radiant energy per unit area or the power radiated per unit area of a perfect blackbody is directly proportional to the fourth power of its absolute temperature.

S.I. unit

[1] Stefan's constant is equal to, $\sigma = 5.67 \times 10^{-8} J m^{-2} s^{-1} K^{-4}$

or

$\frac{J}{m^2 s K^4}$ or $W m^{-2} K^{-4}$

Dimensions:-

$[L^0 M^1 T^{-3} K^{-4}]$

3. KINETIC THEORY OF GASES

Specific heat capacity of gases

1] Principal specific heat:- [C | S]

Amount of heat required to raise the temperature of unit mass of gas through 1° C/1°K is called as principal specific heat.

$$\boxed{S = \frac{dQ}{m \times dT}}$$

$$C = \frac{dQ}{m \times dT}$$

S.I. unit = J/kgK

CGS unit = $\frac{erg}{gm\,K}$

Principal specific heat :-

There are **two** types,

A] Principal specific heat at constant pressure

B] Principal specific heat at constant volume

A] **Principal specific heat at constant pressure:-**

Amount of heat required to raise the temperature of unit mass of gas through 1°C /1°K at constant pressure is known as principal specific heat at constant pressure.

$$\boxed{C_p = \frac{dq}{m \times dT}}$$

It is denoted by Cp.

B] **Principal specific heat at constant volume:-**

Amount of heat required to raise the temperature of unit mass of gas through 1 C /1 K at constant volume is known as principal specific heat at constant volume.

$$\boxed{C_v = \frac{dq}{m \times dT}}$$

It is denoted by Cv.

Molar specific heat:-

Amount of heat required to raise the temperature of 1 mole of gas through 1°C /1°K is called as molar specific heat.

$$\boxed{S = \frac{dq}{n \times dT}}$$

$$\boxed{C = \frac{dq}{n \times dT}}$$

3. KINETIC THEORY OF GASES

S.I. unit is = J / mol K

CGS unit = $\frac{\text{erg}}{\text{mol K}}$

Molar specific heat

There are 2 types,

A] Molar specific heat at constant pressure

B] Molar specific heat at constant volume

A] Molar specific heat at constant pressure:-

Amount of heat required to raise the temperature of 1 mole of gas through 1°C / 1°K at constant pressure is known as molar specific heat at constant pressure.

$$\boxed{C_p = \frac{dq}{n \times dT}}$$

B] Molar specific heat at constant volume:-

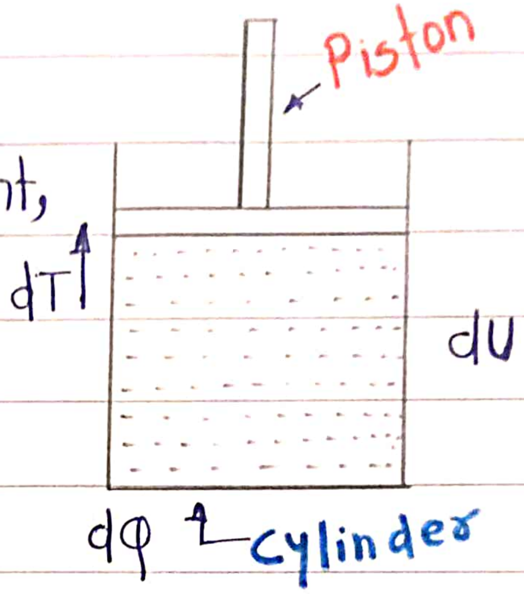
Amount of heat required to raise the temperature of 1 mole of gas through 1° C/ 1°K at constant volume is known as molar specific heat at constant volume.

$$\boxed{C_V = \frac{dq}{n \times dT}}$$

Mayer's Relation :-

Consider 1 mole of ideal gas enclosed in cylinder with air tight, Frictionless & movable piston,

Volume is kept constant,



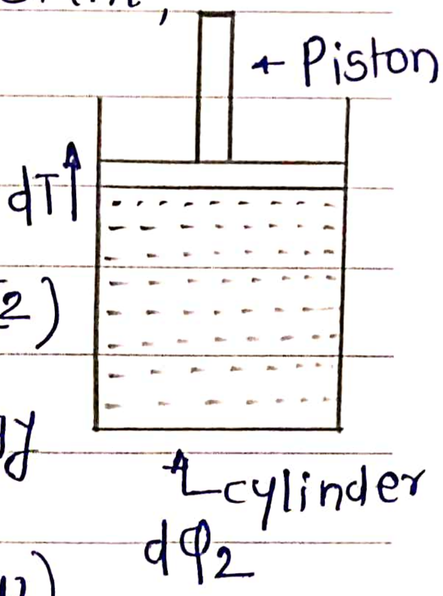
$$dV = 0$$

$$C_V = \frac{d\varphi}{n \times dT}$$

$$C_V = \frac{d\varphi_1}{n \times dT}$$

$$\boxed{d\varphi_1 = C_V \times n \times dT = dU} \quad \cdots (1)$$

When pressure is kept constant,



$$dP = 0$$

$$C_P = \frac{d\varphi_2}{n \times dT}$$

$$\boxed{d\varphi_2 = C_P \times n \times dT} \quad \cdots (2)$$

(i) Increase in internal energy (dU)

(ii) External work done (dw)

$$dQ_2 = dU + dW$$

$$1 \times C_p \times dT = 1 \times C_v \times dT + PdV$$

$$PV = RT$$

$$PdV = RdT$$

$$1 \times C_p \times \cancel{dT} = 1 \times C_v \times \cancel{dT} + R\,\cancel{dT}$$

$$C_p = C_v + R$$

$$\boxed{C_p - C_v = R}$$

This is called mayer's relation.

dQ - calories

dW - Joules

$$\boxed{C_p - C_v = \frac{R}{J}} \quad \text{------} \quad ③$$

Relation between molar specific heat & principal sp. heat

molar sp. heat = (molar mass) x (principal sp. heat)

$$\begin{tikzcd} C \arrow[d] \arrow[rd] & = M_o \times C \arrow[r] \arrow[rd] & C_p \\ C_p & C_v & C_v \end{tikzcd}$$

$C_p = M_o \times C_p$

$C_v = M_o \times C_v$

Put this value in eq^n ③,

$$C_p - C_v = \frac{R}{J}$$

$$M_o \times C_p - M_o \times C_v = \frac{R}{J}$$

$$M_o (C_p - C_v) = \frac{R}{J}$$

$$\boxed{C_p - C_v = \frac{R}{M_o J}}$$