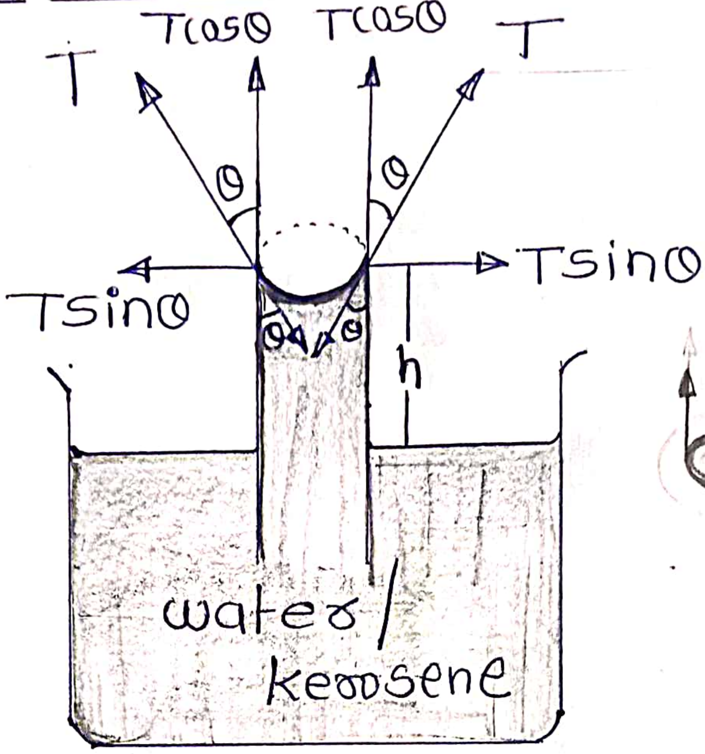
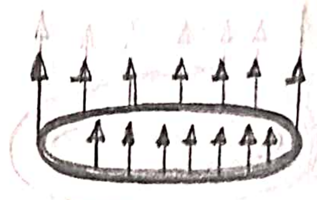
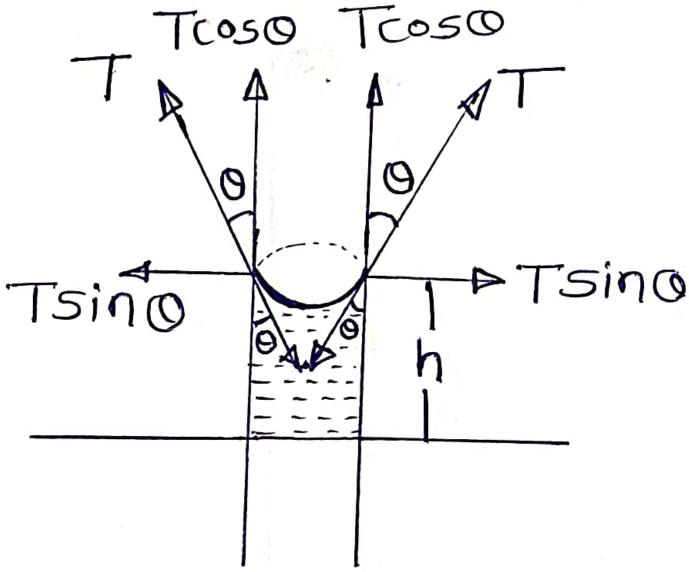
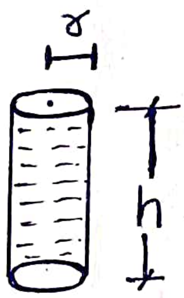
Expression for capillary rise









$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{V}$$

$$m = \rho V$$

$$\underline{\underline{m = \rho \pi r^2 h}}$$

Upward force acting on meniscus $= T\cos\theta \times 2\pi r$ ---- (1)

Downward force = wt. of liquid column

$= m \times g$ (mass of liquid inside capillary)

Downward force $= \rho \pi r^2 h g$ ---- (2)

At equilibrium,

upward force = downward force

$$T\cos\theta \times 2\cancel{\pi}\cancel{r} = \rho \cancel{\pi} r^{\cancel{2}} h g$$

$$2T\cos\theta = \rho r h g$$

$$\boxed{h = \frac{2T\cos\theta}{\rho r g}}$$

where,

ρ - Density of liquid

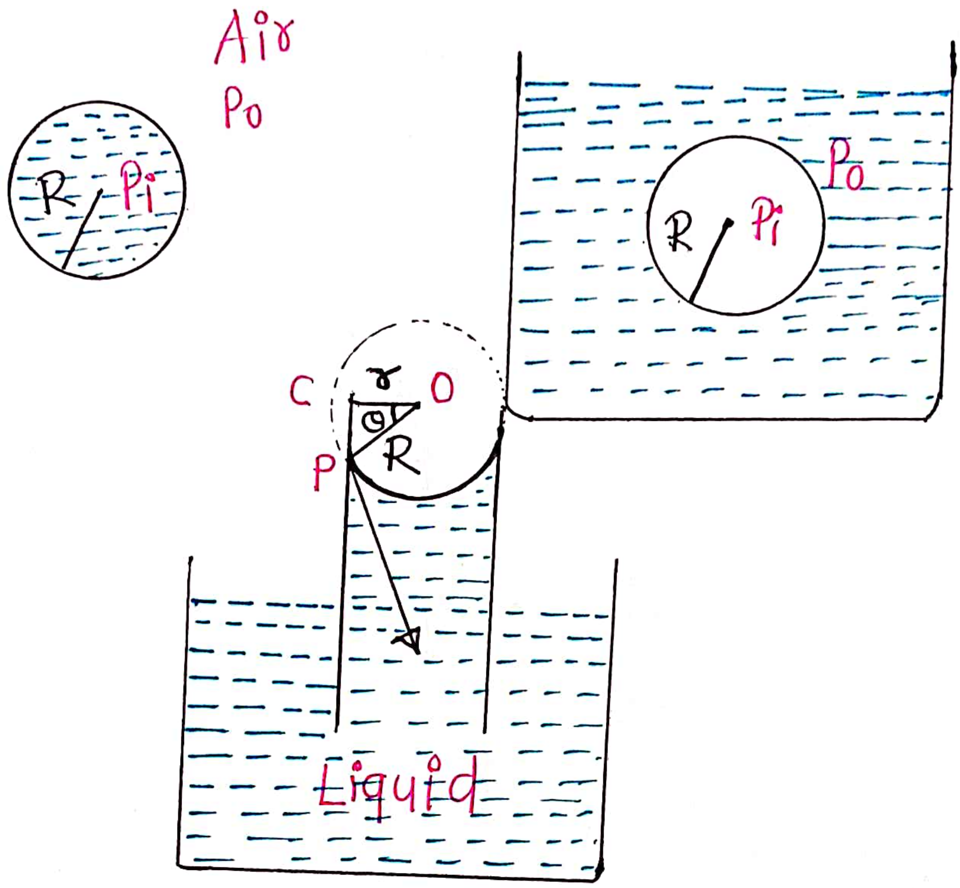
g - Accln due to gravity

r - radius of capillary tube

T - Surface tension of liquid

h - ht. of liquid level

Expression for rise of liquid inside a capillary tube, (Pressure difference)



Excess pressure inside a liquid drop $= \frac{2T}{R}$ ---- (1)

Pressure difference $= h\rho g$ --- (2)

$$h\rho g = \frac{2T}{R} \quad \text{--- (3)}$$

In ΔOCP,

$$\cos\theta = \frac{r}{R}$$

$$R = \frac{r}{\cos\theta}$$

Put, $R = \frac{r}{\cos\theta}$ in eqn (3),

$$h\rho g = \frac{2T}{\frac{r}{\cos\theta}}$$

$$h\rho g = \frac{2T\cos\theta}{r}$$

$$\boxed{h = \frac{2T\cos\theta}{r\rho g}}$$

where,

h = height of liquid level

ρ = density of liquid

g = Accln due to gravity

T = surface tension of liquid

r = radius of capillary tube.

Fluids in Motion

Hydrodynamics :-

The branch of physics which deals with the study of properties of fluids in motion is called hydrodynamics.

Define the following terms,

1] Steady flow.

2] Flow line.

3] Stream line.

4] Flow tube.

1] Steady flow :-

Flow in which measurable property, such as pressure or velocity of the fluid at a given point is constant over time is called steady flow.

2] Flow line :-

Path of an individual particle in a moving fluid is called as flow line.

3] Streamline :-

The curve whose tangent at any point in the flow is in the direction of the velocity of the flow at that point is called streamline.

4] Flow tube :-

An imaginary bundle of flow lines bound by an imaginary wall is called a blow tube.

Critical velocity :- (V_c)

The velocity beyond which a streamline flow becomes turbulent is called critical velocity.

According to Osboone Reynold's, critical velocity is given as,

$$\boxed{V_c = \frac{R_n \eta}{\rho d}}$$

where,

V_c = critical velocity of fluid.

R_n = Reynold's number

η = coefficient of viscosity

ρ = Density of fluid

d = diameter of tube.

Reynold's Number :-

Reynold'd's number is given by,

$$\boxed{R_n = \frac{V_c \rho d}{\eta}}$$

where,

V_c = Critical velocity of the fluid
ρ = Density of liquid
d = diameter of tube
η = coefficient of viscosity.

It has no unit & dimensions.

1) If value of $R_n > 1000$, the flow of liquid is laminar.

2) If value of R_n is greater than 2000, the flow of liquid is turbulent.

3) When R_n lies between 1000 to 2000, then flow becomes unsteady.

Define the following terms,

1) Streamline flow.
2) Turbulent flow.

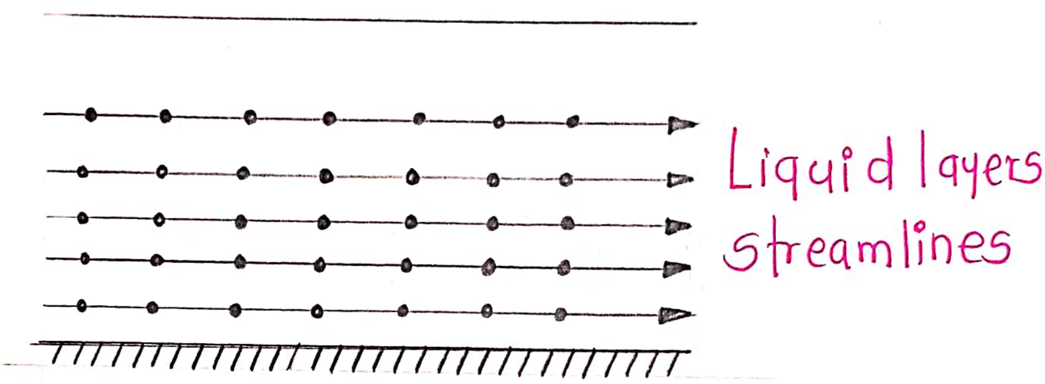
1) streamline flow :-

A steady flow in which adjacent layers of a fluid move smoothly over each other is called streamline or laminar flow.

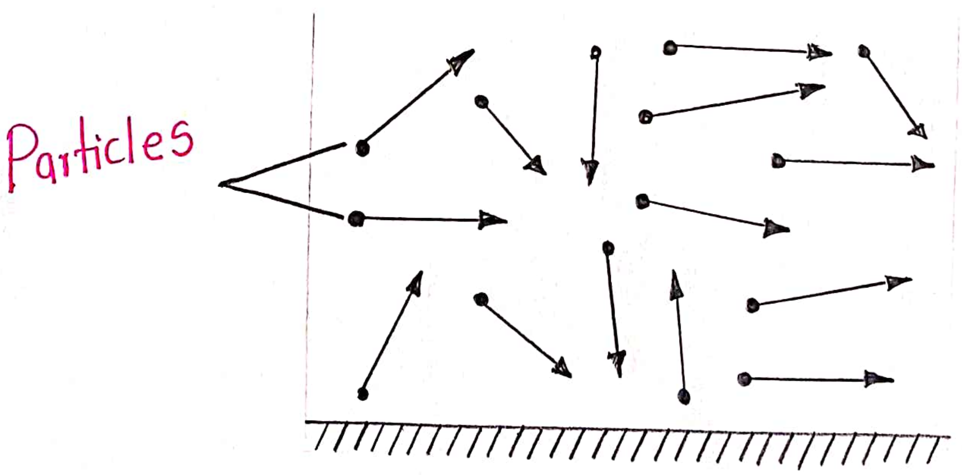
2] Turbulent flow :-

The irregular & unsteady flow of a fluid when its velocity increases beyond critical velocity is known as turbulent flow.

Diagram :-

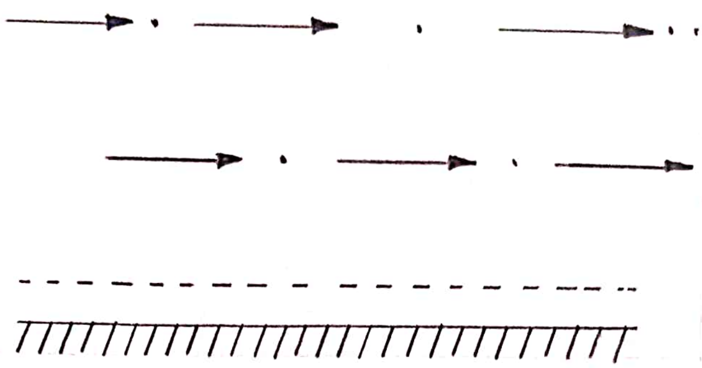


Laminar Flow

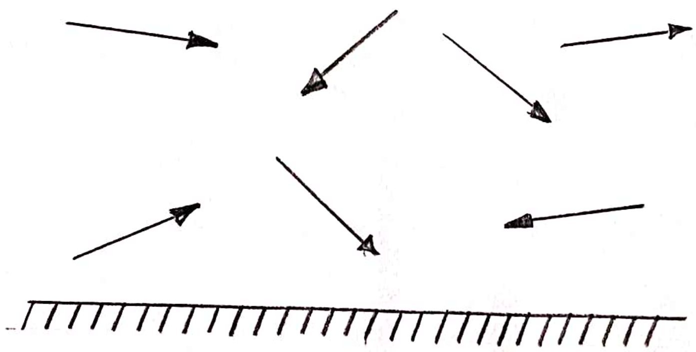


Turbulent Flow

Diagram:-



Irrotational Flow



Rotational Flow

Define

1] Viscosity :-

The property of fluid by virtue of which the relative motion between different layers of a fluid experience a dragging force is called viscosity.

2] Velocity Gradient :-

The rate of change of velocity (dv) with distance (dx) measured from a stationary layer is known as velocity gradient.

$$\boxed{\text{Velocity gradient} = \frac{dv}{dx}}$$

3] Newton's law of viscosity :-

Statement :-

For streamline flow, the viscous force acting on any layers is directly proportional to,

(a) area of the layer (A)

(b) velocity gradient (dv/dx)

Mathematically it is written as,

(a) $F \propto A$ ----(1)

(b) $F \propto \frac{dv}{dx}$ ----(2)

from equation (1) & (2) we have,

$$F \propto A\left(\frac{dv}{dx}\right)$$

$$F = \eta A\left(\frac{dv}{dx}\right)$$

η = constant called coefficient of viscosity

4] Coefficient of viscosity :-

The coefficient of viscosity is defined as the viscous force per unit area per unit velocity gradient.

Unit of η = Ns/m^2 or decapoise (SI)

From Newton's law of viscosity,

$$F = \eta A \frac{dv}{dx}$$

where,

η = coefficient of viscosity

$\frac{dv}{dx}$ = velocity gradient

$$\boxed{\eta = \frac{F}{\frac{dv}{dx} A}}$$

If $A = 1 m^2$

$$\frac{dv}{dx} = 1 s^{-1}$$

$$\boxed{F = \eta}$$

Dimensions of $\eta = [M^1][L^{-1}][T^{-1}]$

$1 Ns/m^2 = 10$ poise

$= 1$ decapoise.

Stoke's law :-

Statement :-

The viscous force acting on a small sphere falling through a medium is directly proportional to the,

Radius of the sphere (r)

its velocity (v) through fluid &

coefficient of viscosity (η) of the fluid.

mathematically,

$$F \alpha \eta r v$$

$$\boxed{F = k\eta r v}$$

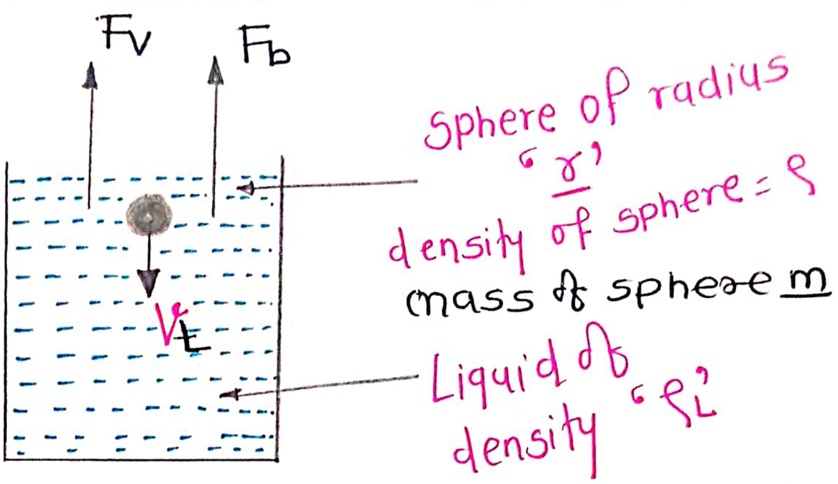
$$\boxed{F = 6\pi\eta r v}$$

where,

K = constant of proportionality

Terminal velocity :-

The constant maximum velocity acquired by a body falling through a viscous liquid is called as terminal velocity.



When a sphere falls through this viscous medium, forces acting on sphere are,

(i) Viscous force Fv acting upward.

(ii) upthrust | buoyant force acting in upward direction.

(iii) Gravitational force = wt. of sphere
$= mg$.

Total upward force = Total downward force

$$F_v + F_b = mg$$

$6\pi\eta r V$ + wt. of liquid displaced by sphere $= mg$

$6\pi\eta r V_t$ + (mass of liquid displaced) $\times g$ = mass of sphere $\times g$

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho_1 = \frac{m}{V} = \rho_1 = \frac{m}{\frac{4}{3}\pi r^3}$$

$$\boxed{m = \rho_1 \frac{4}{3}\pi r^3}$$

$$\text{density} = \frac{\text{mass}}{\text{volume}}$$

$$\rho = \frac{m}{\frac{4}{3}\pi r^3}$$

$$\boxed{m = \rho \times \frac{4}{3}\pi r^3}$$

$$6\pi\eta r V_t + \rho_l \times \frac{4}{3}\pi r^3 \times g = \rho \times \frac{4}{3}\pi r^3 \times g$$

$$6\pi\eta r V_t + \rho_l \times \frac{4}{3}\pi r^3 \times g = \rho \times \frac{4}{3}\pi r^3 \times g$$

$$6\pi\eta r V_t = \rho \times \frac{4}{3}\pi r^3 \times g - \rho_l \times \frac{4}{3}\pi r^3 \times g$$

$$\overset{3}{\cancel{6}}\cancel{\pi}\eta\cancel{r}V_t = \frac{\overset{2}{\cancel{4}}}{3}\cancel{\pi} r^{\overset{2}{\cancel{3}}} g(\rho - \rho_l)$$

$$3\eta V_t = \frac{2}{3} r^2 g(\rho - \rho_l)$$

$$V_t = \frac{2}{3 \times 3} \frac{r^2 g}{\eta}(\rho - \rho_l)$$

$$\boxed{V_t = \frac{2}{9} \frac{r^2 g}{\eta}(\rho - \rho_l)}$$

This is the expression for terminal velocity of the sphere.

Equation of continuity

Statement :-

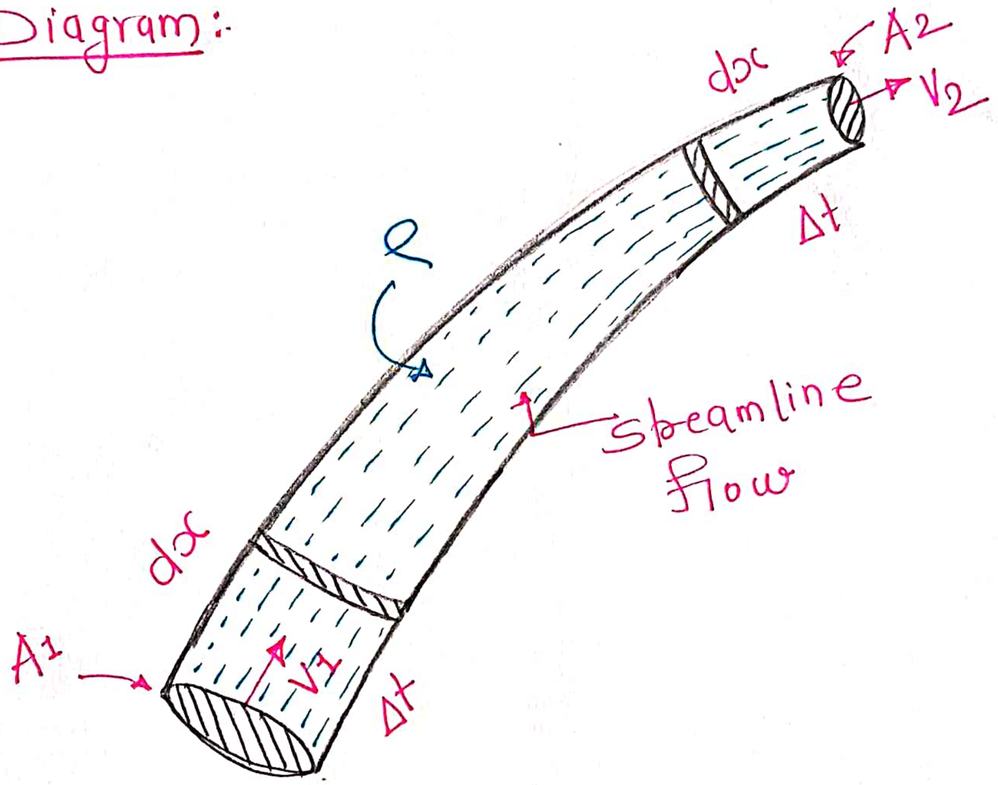
The volume rate of flow of an incompressible fluid for a steady flow is the same throughout the flow.

where,

A = Area of cross section

V = speed of liquid

Diagram :-



mass entering,

$$\Delta m_1 = \rho \underbrace{A_1 V_1 \Delta t}_{\text{Volume}}$$

Mass leaving,

$$\Delta m_2 = \rho \underbrace{A_2 V_2 \Delta t}_{\text{volume}}$$

Mass entering = mass leaving

$$\frac{\Delta m_1}{\Delta t} = \frac{\Delta m_2}{\Delta t}$$

$$\cancel{\rho} A_1 V_1 = \cancel{\rho} A_2 V_2$$

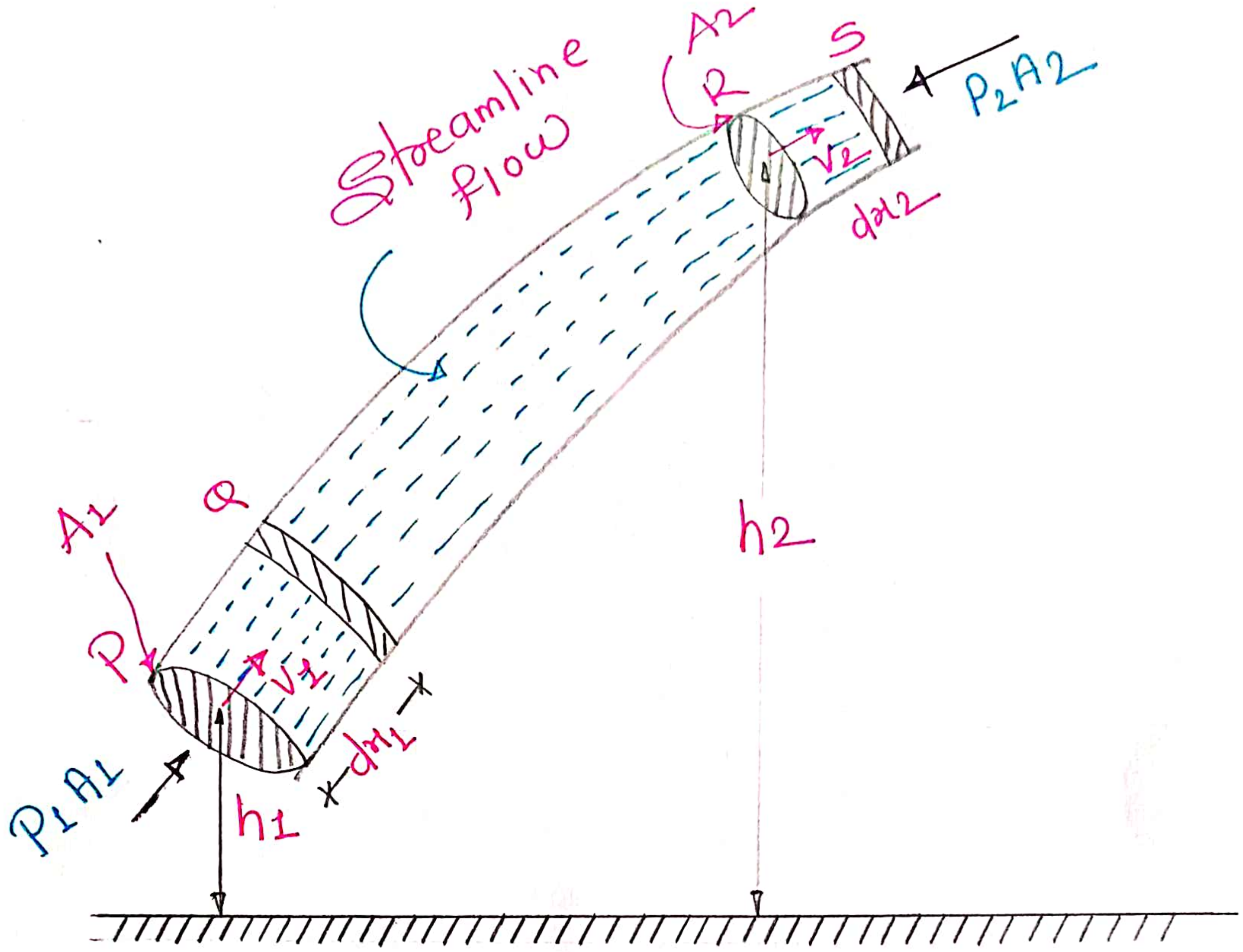
$$\boxed{A_1 V_1 = A_2 V_2}$$

$$\boxed{AV = \text{constant}}$$

Bernoulli's Equation

statement :-

It states that the sum of pressure energy, kinetic energy & potential energy per unit volume of an incompressible non-viscous fluid in streamline, irrotational flow remains constant.



where,

V_1, V_2 = speed of the fluid at lower end & upper end

A_1, A_2 = Area of cross section at the lower end & upper end

P_1, P_2 = Pressure of the fluid at lower end & upper end

dx_1, dx_2 = distance travelled by the fluid at lower end & upper end.

The volume dV of the fluid,

$$dV = A_1 dx_1 = A_2 dx_2 \quad ---- (1)$$

The net work done,

$$W = P_1 A_1 dx_1 - P_2 A_2 dx_2 \quad -- (2)$$

Putting eqⁿ (1) in (2),

$$W = P_1 dV - P_2 dV$$

$$\boxed{W = dV(P_1 - P_2)} \quad --- (3)$$

Now,

mass of the fluid between P & Q,

$$\text{Density} = \frac{\text{mass}}{\text{volume}}$$

$$\text{mass} = \text{Density} \times \text{volume}$$

$$m = \rho \times V$$

$$\boxed{m = \rho \times A_1 dx_1}$$

kinetic energy of the fluid between P & Q,

$$K.E. = \frac{1}{2} m v^2$$

$$K.E. = \frac{1}{2} \rho A_1 dx_1 V_1^2 \quad -- \quad (m = \rho A_1 dx_1)$$

Potential energy of the fluid between P & Q,

$$P.E. = mgh$$

$$P.E. = \rho A_1 dx_1 g h_1$$

$$\boxed{P.E. = \rho\, dv\, g h_1}$$

mass of the fluid between R & S,

$$m = \rho \times V$$

$$\boxed{m = \rho \times A_2 dx_2}$$

kinetic eneogy of the fluid between R&S,

$$K.E. = \frac{1}{2} m v^2$$

$$K.E. = \frac{1}{2} \rho A_2 dx_2 V_2^2$$

$$K.E. = \frac{1}{2} \rho\, dV\, V_2^2$$

Potential eneogy of the fluid between R & S,

$$P.E. = mgh$$

$$P.E. = \rho A_2 dx_2 g \times h_2$$

$$\boxed{P.E. = \rho\, dv\, g h_2}$$

The net change in kinetic eneogy,

$$\Delta K.E. = \frac{1}{2} \rho (A_2 dx_2) V_2^2 - \frac{1}{2} \rho (A_1 dx_1) V_1^2$$

$$\Delta K.E. = \frac{1}{2} \rho\, dV\, V_2^2 - \frac{1}{2} \rho\, dV\, V_1^2$$

$$\boxed{\Delta K.E. = \frac{1}{2} \rho\, dv\, (V_2^2 - V_1^2)} \quad \text{---} (4)$$

The net change in gravitational potential energy,

$$\Delta P.E. = \rho dv g h_2 - \rho dv g h_1$$

$$\boxed{\Delta P.E. = \rho dv g (h_2 - h_1)} \quad \cdots (5)$$

The total mechanical energy,

$$W = \Delta K.E. + \Delta P.E.$$

Substitute eq^n (3), (4) & (5) in above equation,

$$(P_1 - P_2)\cancel{dv} = \frac{1}{2}\rho \cancel{dv}(v_2^2 - v_1^2) + \rho \cancel{dv} g (h_2 - h_1)$$

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) + \rho g(h_2 - h_1)$$

This is known as Bernoulli's equation

Bernoulli's eq^n can be written as,

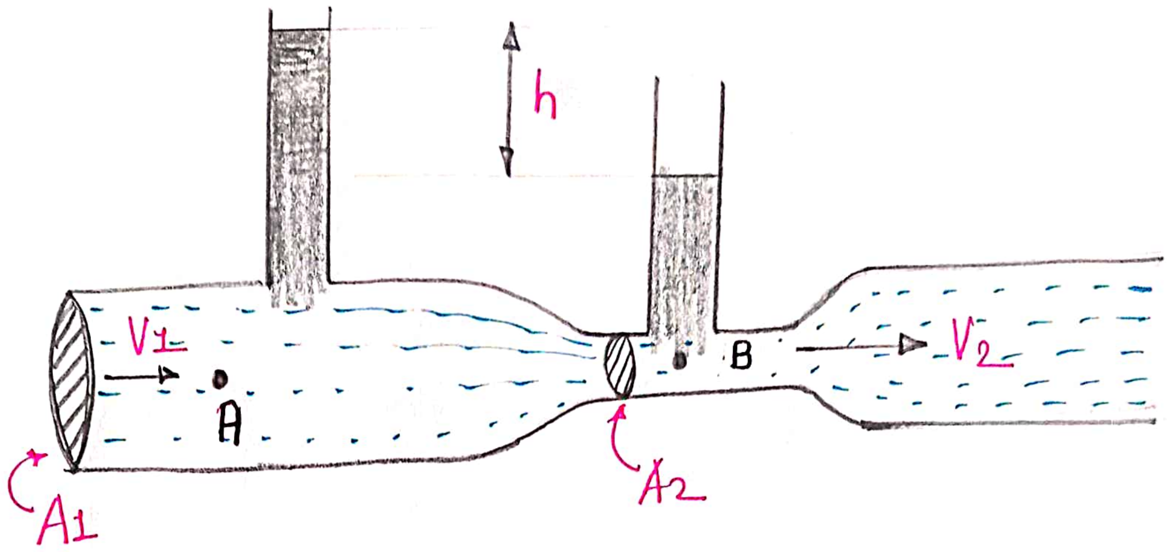
$$P_1 - P_2 = \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 + \rho g h_2 - \rho g h_1$$

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

or

$$P + \frac{1}{2}\rho v^2 + \rho g h = \text{constant}$$

Venturi-Tube (Venturimeter)



The venturi-tube is a device which is used to measure the flow speed of incompressible fluid.

Apply Bernoulli's equation,

$$P_A + \frac{1}{2}\rho V_1^2 + \rho g h_1 = P_B + \frac{1}{2}\rho V_2^2 + \rho g h_2$$

At the same level,

$$P_A + \frac{1}{2}\rho V_1^2 = P_B + \frac{1}{2}\rho V_2^2$$

$$P_A - P_B = \frac{1}{2}\rho V_2^2 - \frac{1}{2}\rho V_1^2$$

$$P_A - P_B = \frac{1}{2}\rho (V_2^2 - V_1^2)$$

Now,

Equation of continuity,

$$A_1 V_1 = A_2 V_2$$

$$\boxed{V_2 = \frac{A_1}{A_2} V_1}$$

$$P_A - P_B = \frac{1}{2}\rho\left(\frac{A_1^2}{A_2^2} V_1^2 - V_1^2\right)$$

$$P_A - P_B = \frac{1}{2}\rho V_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$\rho g h = \frac{1}{2}\rho V_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$2gh\cancel{\rho} = \cancel{\rho} V_1^2\left(\frac{A_1^2}{A_2^2} - 1\right)$$

$$V_1^2 = \frac{2gh}{\left(\frac{A_1^2}{A_2^2} - 1\right)}$$

$$V_1 = \left[\frac{2gh}{\left(\frac{A_1^2}{A_2^2} - 1\right)}\right]^{1/2}.$$