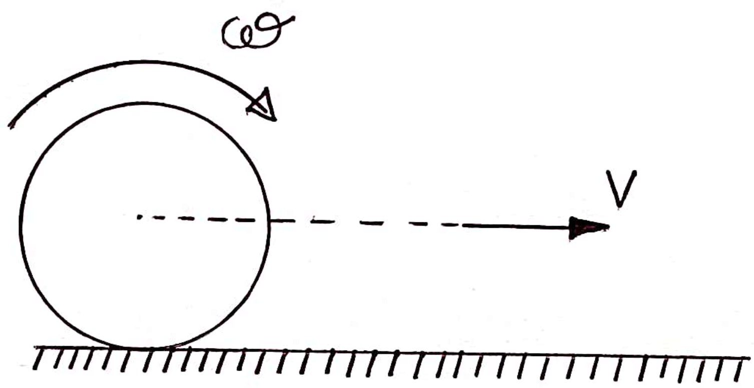
Kinetic Energy of a Rolling Body

Rolling motion :-

A motion is said to be rolling motion if a body posses both i.e linear motion (translational motion) & rotational motion.



$[K.E.]_{translational}$,

$$\boxed{E_{trans} = \frac{1}{2} M v^2} \quad \text{---- (1)}$$

$[K.E]_{rotational}$,

$$\boxed{E_{rot} = \frac{1}{2} I \omega^2} \quad \text{----- (2)}$$

To calculate,

[K.E.] rolling,

$$K.E._{rolling} = K.E._{Translational} + K.E._{rotational}$$

$$\boxed{[K.E.]_{roll} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2}$$

<u>In terms of linear velocity (v)</u>

$$[K.E.]_{rolling} = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$$

$$\boxed{I = Mk^2} \; ; \; V = \omega R$$

$$\boxed{\omega = \frac{V}{R}}$$ Put in above eqn

$$[K.E.]_{rolling} = \frac{1}{2}Mv^2 + \frac{1}{2}Mk^2\left(\frac{V}{R}\right)^2$$

$$= \frac{1}{2}Mv^2 + \frac{1}{2}Mk^2 \times \frac{V^2}{R^2}$$

$$\boxed{[K.E]_{rolling} = \frac{1}{2}MV^2\left[1 + \frac{k^2}{R^2}\right]}$$

In terms of angular velocity (ω)

$$[K.E.]_{rolling} = \frac{1}{2} M v^2 \left[1 + \frac{k^2}{R^2}\right]$$

$$= \frac{1}{2} M (R \cdot \omega)^2 \left[1 + \frac{k^2}{R^2}\right]$$

$$[K.E.]_{rolling} = \frac{1}{2} M R^2 \omega^2 \left[1 + \frac{k^2}{R^2}\right]$$

$$= \frac{1}{2} M \cancel{R^2} \omega^2 \left[\frac{R^2 + k^2}{\cancel{R^2}}\right]$$

$$\boxed{[K.E]_{rolling} = \frac{1}{2} M \omega^2 [R^2 + k^2]}$$